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Robust regression type estimators to determine the population mean under simple and two-stage random sampling techniques

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For the estimation of population mean, there are several ratio and regression type estimators available in literature. However, they can be misleading to contain the desired results when data are contaminated by outliers. In recent past, researchers provided the solution of this issue by utilizing some robust regression tools and develop a class of ratio type estimators under simple random sampling scheme. Further they extended the work using ratio technique. In this paper, we proposed a new class of robust regression type estimators with utilizing LAD, LMS, LTS, Huber-M, Hampel-M, Tukey-M, Huber-MM as robust regression tools. The desired class is subsequently extended for two stage sampling, where mean of the study variable is not available at first stage. Also, we have developed some reviewed and proposed estimators under above mentioned sampling technique. Further, we have divided our supposition into two cases as: (i)- when drawn a second stage sample depends upon first stage sample and, (ii)- when drawn a second stage sample is independent of first stage sample. The mean square expressions of the proposed estimators have been determined through Taylor series expansion. A real life application and the simulation study are also provided

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to assess existing and proposed estimators. In the light of numerical illustration, we see that our proposed estimators give more efficient results than the reviewed ones.

keywords: Regression-type estimators, robust regression tools, simple random sampling, two stage sampling.

1 Introduction

Now a days, a widely utilized phrase that we are living in the age of information. Utilizing this phrase, we are not just featuring the volume and speed of existing information yet in addition underlining the need of its exact stream. The later part of the above comprehension is legitimately connected with the true intention of the urge of gathering information. The intention is to empower ourselves of absolutely profiling our environment and in this way supporting the optimal decision making process. In fulfilling the need of multidisciplinary request interlocking government issues, business basic leadership, clinical examinations and mental profiling and so on. It is of nothing unexpected if the sampling theory and method remains at the core of applied research literature. One of the most significant goal of practices in sampling stays with the estimation of mean for study variable. To meet this challenge of achieving more precise estimate of population mean, ratio method of estimation is the highly praised way utilizing supplementary information. Laplace in eighteenth century, as an early client/user of supplementary information in the estimation of total population of France, gave the method of utilization of supplementary information in an efficient way. Specifically, he referenced, "The register of births, which are kept with care in order to assure the condition of the citizens, can serve to determine the population of great empire without resorting a census of its inhabitants. But for this it is necessary to know the ratio of the population to the annual birth", see for example, Lohr (1999).

A vast amount of literature is available on ratio-type and regression-type estimators. Such as Oral and Oral (2011), Koyuncu (2012), Abid et al. (2016a,b), Shahzad et al. (2018), Hanif and Shahzad (2019), Bulut and Zaman (2019), Naz et al. (2019) and Irfan et al. (2019) have suggested several classes of estimators for simple random sampling with utilizing supplementary information. For more about ratio estimators, we refer Jemain et al. (2008), Al-Omari et al. (2008), Al-Omari et al. (2009), Al-Omari and Jaber (2010), Al-Omari (2012), Al-Omari and Bouza (2015), Bouza et al. (2017), Al-Omari and Al-Nasser (2018). For the estimation of population mean, ratio (product) estimators are better if correlation is positive (negative) between auxiliary and study variables. The conventional regression estimators solve the issue related to the sign (positive/negative) of correlation and provide better results than the traditional ratio or product type estimators. Note that conventional regression-type estimators are dependent on conventional regression coefficient, i.e. known as ordinary least square (OLS) regression coefficient. However, the estimates by OLS become inappropriate when outliers exist in data. For solving this issue, Kadilar et al. (2007) incorporated Huber-M robust regression technique instead of

OLS. After that Zaman and Bulut (2019a) extended the idea of Kadilar et al. (2007) and developed a class of ratio-type-estimators with utilizing some other robust-regression-tools, such as: least absolute deviations (LAD), least trimmed squares (LTS), least median of squares (LMS), Hampel-M, Tukey-M and Huber-MM.

The basic purposes of LAD and LMS are to minimize the absolute residuals and median of squared residuals, respectively. The squared errors are arranged in LTS method and OLS is run by utilizing observations based on the first (smallest) z errors. The theme of M estimation is to minimize the q functions that are satisfied under some conditions Zaman and Bulut (2019a). There are many q functions are available in literature, see for examples, Huber (1964, 1973), Hample (1971) and Tukey (1977). For more statistical efficiency and breakdown point, Yohai (1987) presented MM robust regression tool. For more about these estimation tools, we refer for the interested readers to Zaman and Bulut (2020) and Ali et al. (2019). Moreover, these techniques have been extended for stratified random sampling scheme by Zaman and Bulut (2019a) and Zaman (2019) developed another class of estimators in the same context and achieved the results equivalent to traditional regression estimator. In this work, taking inspiration from Zaman and Bulut (2020) and Zaman (2019), we introduce a new and improved class of robust-regression-type estimators for the mean estimation, when study variable contaminated by outliers. Outliers are the observations in a data set which appear to be inconsistent with the rest of that data set. Presence of outliers significantly effect mean estimation which is one of the most important measure of central tendency.

Mean estimators utilizing traditional regression coefficient are being mostly used for the estimation of population mean, i.e. 'Y'. However, outliers may have significant impact on the traditional regression coefficient calculated from OLS tool. Hence the estimate of population mean i.e. (y), based upon OLS may indicate poor performance. Kadilar et al. (2007) and Zaman and Bulut (2019a) provided the solution of this issue by incorporating robust regression coefficients in this context. Robust regression is used when OLS assumptions are violated. In such circumstances, the robust-regression tools provide better results because outliers are assigned with lower weight. Zaman and Bulut (2019a) introduced the following class of estimators utilizing robust regression tools for the estimation of mean as given in the following form:

$$\bar{y}_{zb_i} = \frac{\bar{y} + b_{(i)}(\bar{X} - \bar{x})}{c\bar{x} + d}(c\bar{X} + d), \qquad i = 1, 2, \cdots, 35.$$

In the above expression, there are $\bar{x}$ and $\bar{X}$ are sample and population means of auxiliary variable, whereas $\bar{y}$ be the sample mean of study variable. The variances of these unbiased sample means, $(\bar{x}, \bar{y})$ are $V(\bar{x}) = \theta S_x^2$ and $V(\bar{y}) = \theta S_y^2$. Further, c and d are either zero or one or some known population measures namely, the coefficient of variation (C_x), the coefficient of kurtosis $(\beta_2(x))$ and the robust regression coefficients $(b_{(i)})$. We have provided a list of family members of $\bar{y}_{zb_i}$ in Table 1.

MSE of Zaman and Bulut (2019a) family of estimators is given below

$$MSE(\bar{y}_{zb_i}) = \theta\big(S_y^2 + g_i^2 S_x^2 + 2B_i g_i S_x^2 + B_i^2 S_x^2 - 2g_i S_{xy} - 2B_i S_{xy}\big) \quad for \quad i = 1, 2, ..., 35, \tag{1}$$

where, $g_i = \dfrac{c\bar{Y}}{c\bar{X}+d}$ and $\theta = (\dfrac{1-f}{n})$ for i=1,2,...,35. Further, S_y^2 and S_x^2 are the unbiased variances of Y and X, respectively. Note that $\bar{y}_{zb_{16}} - \bar{y}_{zb_{20}}$ belongs to Kadilar et al. (2007) in Table 1.

Zaman (2019) introduced the following class of estimators for mean estimation, whose utilizing robust regression tools as:

$$\bar{y}_{z_i} = k\frac{\bar{y}+b_{(i)}(\bar{X}-\bar{x})}{(c\bar{x}+d)}(c\bar{X}+d) + (1-k)\frac{\bar{y}+b_{(i)}(\bar{X}-\bar{x})}{(c\bar{x}+d)}(c\bar{X}+d),$$

where k is a constant such that it provides the minimum $MSE(\bar{y}_{z_i})$. The MSE of $\bar{y}_{z_i}$ is as follows

$$MSE(\bar{y}_{z_i}) = \theta\big(S_y^2 - 2\delta S_{yx} + \delta^2 S_x^2\big), \tag{2}$$

where $\delta = \big(k(B_{(i)}+g_1) + (1-k)(B_{(i)}+g_i)\big)$.
Zaman (2019) replaced $(\delta = B)$ in above MSE expression, and get minimum MSE of $\bar{y}_{z_i}$ as follows

$$MSE(\bar{y}_{z_i}) = \theta S_y^2(1-\rho^2), \tag{3}$$

which is the MSE of traditional regression estimator, i.e. $\bar{y}_{reg} = \bar{y} + b_{(i)}(\bar{X}-\bar{x})$.

The rest of the manuscript is constructed as follows: In Section 2.1, we have proposed a new class of robust-regression-type estimators. The theoretical mean squared error (MSE) of proposed class is also derived. Section 2.2 has been dedicated to two stage sampling scheme. We also introduced the reviewed and proposed estimators under two stage sampling scheme with their theoretical MSE expressions in Section 2.2. Results and discussion are provided in Section 3. The manuscript is ended with some concluding remarks in Section 4.

2 The proposed estimators

2.1 The robust-regression-type-estimators

Taking motivation from Zaman and Bulut (2019a) and Zaman (2019), we propose the following class of estimators as given below:

$$\bar{y}_{N_i} = k_1\bigg(\bar{y} + b_{(i)}(\bar{X}-\bar{x})\bigg) + k_2\left(\bar{X}-\bar{x}\right) \quad for \quad i = 1,2,...,7 \tag{4}$$

where, k_1 and k_2 are real constants. Further $\bar{x}$, $\bar{y}$ and $b_{(i)}$ have there usual meanings as defined in Sect. 2. A complete list of family members of proposed techniques is available in Table 2. To obtain the MSE of (3.1), let us define $\bar{y} = (1+\eta_o)\bar{Y}$ and $\bar{x} = (1+\eta_1)\bar{X}$. Utilizing these notations η_i $(i = 0,1)$, we can write $E(\eta_o) = E(\eta_1) = 0$, $E(\eta_o^2) = \theta C_y^2$, $E(\eta_1^2) = \theta C_x^2$ and $E(\eta_o\eta_1) = \theta C_{yx}$. Now expending $\bar{y}_{N_i}$ in terms of η_o and η_1 as

$$\bar{y}_{N_i} = k_1\bar{Y}\bigg(1 + \eta_o - R'b_{(i)}\eta_1\bigg) - k_2\bar{X}\eta_1.$$

$$\bar{y}_{N_i} - \bar{Y} = k_1\bar{Y}\left(1 + \eta_o - R'b_{(i)}\eta_1\right) - k_2\bar{X}\eta_1 - \bar{Y}. \tag{5}$$

By taking square of (5), ignoring higher order terms and applying expectation, the MSE of $\bar{y}_{N_i}$ is given below

$$MSE(\bar{y}_{N_i}) = \bar{Y}^2 + k_1^2\Phi_{A_N} + k_2^2\Phi_{B_N} + 2k_1k_2\Phi_{C_N} - 2k_1\Phi_{D_N}, \tag{6}$$

where

$$\begin{aligned}
\Phi_{A_N} &= \bar{Y}^2\left(1 + \theta\left(C_y^2 + R^{'}b_{(i)}\left(R^{'}b_{(i)}C_x^2 - 2C_{yx}\right)\right)\right),\\
\Phi_{B_N} &= \theta\bar{X}^2C_x^2,\\
\Phi_{C_N} &= \theta\bar{X}\bar{Y}[R^{'}b_{(i)}C_x^2 - 2C_{yx}],\\
\Phi_{D_N} &= \bar{Y}^2,\\
R^{'} &= \frac{\bar{X}}{\bar{Y}}.
\end{aligned}$$

The MSE is minimized when

$$k_1^{opt} = \left[\frac{\Phi_{B_N}\Phi_{D_N}}{\Phi_{A_N}\Phi_{B_N} - \Phi_{C_N}^2}\right],$$

and

$$k_2^{opt} = \left[-\frac{\Phi_{C_N}\Phi_{D_N}}{\Phi_{A_N}\Phi_{B_N} - \Phi_{C_N}^2}\right].$$

By substituting k_1^{opt} and k_2^{opt} in (6), we get minimum MSE of $\bar{y}_{N_i}$ as given below

$$MSE_{min}(\bar{y}_{N_i}) = \left[\bar{Y}^2 - \frac{\Phi_{B_N}\Phi_{D_N}^2}{\Phi_{A_N}\Phi_{B_N} - \Phi_{C_N}^2}\right]. \tag{7}$$

Remarks 3.1

- By replacing $(k_1 = 1, b_{(i)} = 0, k_2 = 0)$, $\bar{y}_{N_i}$ becomes unbiased mean estimator.
- By replacing $(k_1 = 1, b_{(i)} = b_{(i)}, k_2 = 0)$, $\bar{y}_{N_i}$ becomes regression estimator, and will be equally important as $\bar{y}_{z_i}$ or $\bar{y}_{reg}$.
- In light of above two points, we can say that $\bar{y}$, $\bar{y}_{reg}$, and $\bar{y}_{z_i}$, are the special cases of $\bar{y}_{N_i}$.

2.2 Two stage sampling scheme

Whenever, if the desired information about of supplementary variable does not available, a two-stage sampling plan is utilized for acquiring the improved estimator because of its financially savvy and simplicity. Neyman (1983) was the first who gave the idea of two-stage sampling in evaluating the population parameters. This sampling plan is utilized to get the information about supplementary variable efficiently by selecting a greater sample from the initial or first stage and moderate size sample (comparatively small sample as compare to first stage) at the second stage. Sukhatme (1962) utilized two-stage inspecting plan to develop a family of ratio-type-estimators. For more details about two-stage sampling, interested readers may refer to Cochran (1977).

Under two-stage sampling plan, we select a first stage sample of size n_1 units from the population of size N using SRSWOR plan. After that we select a second stage sample of size n_2. It is worth mentioning that we are considering two cases for second stage sample as follows:
Case I: The second stage sample, n_2 is a sub-sample of the first stage sample, n_1.
Case II: The second stage sample, n_2 is independent of the first stage sample i.e. n_1.
For more details about these cases, interested readers may refer to Zaman and Kadilar (2021).

2.2.1 Two stage sampling scheme: adapted estimators

In this sub-section, we are adapting Kadilar et al. (2007) and Zaman and Bulut (2019a) family of estimators under two-stage sampling scheme as given by

$$\bar{y}_{zb_i}^{'} = \frac{\bar{y}_2 + b_{(i)}(\bar{x}_1 - \bar{x}_2)}{(c\bar{x}_2 + d)}(c\bar{x}_1 + d) \quad for \quad i = 1, 2, ..., 35.$$

In the above expression, $(\bar{x}_2, \bar{y}_2)$ representing sample means at second stage and $\bar{x}_1$ be the sample mean at first stage. Further, c and d have the same meanings as described in previous section. The family members of $\bar{y}_{zb_i}^{'}$ are same as $\bar{y}_{zb_i}$, available in Table 1. Zaman and Bulut (2019a) have used Taylor series method for $h(\bar{y}, \bar{x}) = \bar{y}_{zb_i}$, and obtained theoretical MSE. In current section, we are adapting their methodology for $h(\bar{y}_2, \bar{x}_1, \bar{x}_2) = \bar{y}_{zb_i}^{'}$, and obtaining MSE for case-I as follows:

$$MSE(\bar{y}_{zb_i}^{'})_I = d\Sigma d^{'}, \tag{8}$$

where

$$d = \left[\frac{\delta h(\bar{y}_2, \bar{x}_1, \bar{x}_2)}{\delta \bar{y}_2} |_{\bar{Y},\bar{X}} \quad \frac{\delta h(\bar{y}_2, \bar{x}_1, \bar{x}_2)}{\delta \bar{x}_1} |_{\bar{Y},\bar{X}} \quad \frac{\delta h(\bar{y}_2, \bar{x}_1, \bar{x}_2)}{\delta \bar{x}_2} |_{\bar{Y},\bar{X}} \right],$$

$$d = [1 \quad (g_i + B_i) \quad -(g_i + B_i)] .$$

$$\sum = \begin{bmatrix} V(\bar{y}_2) & Cov(\bar{y}_2, \bar{x}_1) & Cov(\bar{y}_2, \bar{x}_2) \\ Cov(\bar{x}_1, \bar{y}_2) & V(\bar{x}_1) & Cov(\bar{x}_1, \bar{x}_2) \\ Cov(\bar{x}_2, \bar{y}_2) & Cov(\bar{x}_2, \bar{x}_1) & V(\bar{x}_2) \end{bmatrix}$$

where

$$V(\bar{y}_2) = \gamma_2 S_y^2,$$

$$V(\bar{x}_1) = \gamma_1 S_x^2,$$

$$V(\bar{x}_2) = \gamma_2 S_y^2,$$

$$Cov(\bar{y}_2, \bar{x}_1) = Cov(\bar{x}_1, \bar{y}_2) = \gamma_1 S_{yx},$$

$$Cov(\bar{y}_2, \bar{x}_2) = Cov(\bar{x}_2, \bar{y}_2) = \gamma_2 S_{yx},$$

$$Cov(\bar{x}_1, \bar{x}_2) = Cov(\bar{x}_2, \bar{x}_1) = \gamma_1 S_x^2.$$

Utilizing these defined notations of variances and co-variances, and hence substituting the values of d and Σ in (8), MSE expressions of $\bar{y}'_{zb_i}$ for case-I as follows:

$$MSE(\bar{y}'_{zb_i})_I = \gamma_2 S_y^2 + (\gamma_2 - \gamma_1)\bigg((g_i + B_i)^2 S_x^2 - 2(g_i + B_i)S_{yx}\bigg).$$

To obtain MSE for case-II, all the notations will remain same except, $Cov(\bar{y}_2, \bar{x}_1) = Cov(\bar{x}_1, \bar{y}_2) = 0$ and $Cov(\bar{x}_1, \bar{x}_2) = Cov(\bar{x}_2, \bar{x}_1) = 0$. Hence the MSE of $\bar{y}'_{zb_i}$ for case-II is given by

$$MSE(\bar{y}'_{zb_i})_{II} = \gamma_2(S_y^2 - 2(g_i + B_i)S_{yx}) + (g_i + B_i)^2(\gamma_1 + \gamma_2)S_x^2.$$

As the minimum MSE of Zaman (2019) class of estimators is equal to traditional regression estimator. So we are considering here traditional regression estimator for two phase sampling as follows

$$\bar{y}'_{reg} = \bar{y}_2 + b_{(i)}(\bar{x}_1 - \bar{x}_2).$$

Note that, Pradhan (2005) only provide MSE expressions for $\bar{y}'_{reg}$ case-II. So, we incorporated their MSE expressions for case-II. We also find the MSE expressions for $\bar{y}'_{reg}$ case-I.
The MSE of $\bar{y}'_{reg}$ for case-I and case-II respectively, as given below

$$MSE(\bar{y}'_{reg})_I = S_y^2[\gamma_2 - (\gamma_2 - \gamma_1)\rho^2],$$

$$MSE(\bar{y}'_{reg})_{II} = S_y^2[\gamma_2 + (\gamma_2 - \gamma_1)\rho^2],$$

where $\gamma_1 = \left(\dfrac{1}{n_1} - \dfrac{1}{N}\right)$ and $\gamma_2 = \left(\dfrac{1}{n_2} - \dfrac{1}{N}\right)$.

2.2.2 Two stage sampling scheme: proposed estimators

In current sub-section, we are presenting proposed class of estimators under two-stage sampling scheme as given bellow

$$\bar{y}'_{N_i} = k_1\left\{\bar{y}_2 + b_{(i)}(\bar{x}_1 - \bar{x}_2)\right\} + k_2\left(\bar{x}_1 - \bar{x}_2\right) \quad for \quad i = 1, 2, ..., 7$$

The family members of $\bar{y}'_{N_i}$ are same as $\bar{y}_{N_i}$, available in Table 2. To obtain MSE for case-I, lets us define $\eta_{y2} = \dfrac{\bar{y}_2 - \bar{Y}}{\bar{Y}}$, $\eta_{x_1} = \dfrac{\bar{x}_1 - \bar{X}}{\bar{X}}$ and $\eta_{x_2} = \dfrac{\bar{x}_2 - \bar{X}}{\bar{X}}$.
Utilizing these notations, we can write $E(\eta_{y_2}) = E(\eta_{x_1}) = E(\eta_{x_2}) = 0$, $E(\eta^2_{y_2}) = \gamma_2 C^2_y$, $E(\eta^2_{x_1}) = \gamma_1 C^2_x$, $E(\eta^2_{x_2}) = \gamma_2 C^2_x$, $E(\eta_{y_2}\eta_{x_1}) = \gamma_1 C_{yx}$, and $E(\eta_{y_2}\eta_{x_2}) = \gamma_2 C_{yx}$, and $E(\eta_{x_1}\eta_{x_2}) = \gamma_1 C^2_x$. Now, expending $\bar{y}'_{N_i}$ in terms of $\eta' s$ as given below:

$$\bar{y}'_{N_i} = k_1\{\bar{Y}(1+\eta_{y_2}) + b_{(i)}\bar{X}(\eta_{x_1} - \eta_{x_2})\} + k_2\bar{X}(\eta_{x_1} - \eta_{x_2}).$$

$$\bar{y}'_{N_i} - \bar{Y} = k_1\{\bar{Y}(1+\eta_{y_2}) + b_{(i)}\bar{X}(\eta_{x_1} - \eta_{x_2})\} + k_2\bar{X}(\eta_{x_1} - \eta_{x_2}) - \bar{Y}. \tag{9}$$

Squaring (9), applying expectation, we get MSE of the estimator $\bar{y}'_{N_i}$ upto the order n^{-1}, as

$$MSE(\bar{y}'_{N_i})_I = \bar{Y}^2 + k_1^2\tau_{A_N} + k_2^2\tau_{B_N} + 2k_1k_2\tau_{C_N} - 2k_1\tau_{D_N}, \tag{10}$$

where

$$\begin{aligned}
\tau_{A_N} &= \left[\bar{Y}^2(1+\gamma_2 C_y^2) + (\gamma_2 - \gamma_1)b_{(i)}\bar{X}\left\{b_{(i)}\bar{X}C_x^2 - 2\bar{Y}C_{yx}\right\}\right],\\
\tau_{B_N} &= \bar{X}^2(\gamma_2 - \gamma_1)C_x^2,\\
\tau_{C_N} &= (\gamma_1 - \gamma_2)\bar{X}[\bar{Y}C_{yx} - b_{(i)}\bar{X}C_x^2],\\
\tau_{D_N} &= \bar{Y}^2.
\end{aligned}$$

Which is minimum for

$$k_1^{opt} = \left[\frac{\tau_{B_N}\tau_{D_N}}{\tau_{A_N}\tau_{B_N} - \tau^2_{C_N}}\right],$$

and

$$k_2^{opt} = \left[-\frac{\tau_{C_N}\tau_{D_N}}{\tau_{A_N}\tau_{B_N} - \tau^2_{C_N}}\right].$$

$$MSE(\bar{y}'_{N_i})_I = \left[\bar{Y}^2 - \frac{\tau_{B_N}\tau^2_{D_N}}{\tau_{A_N}\tau_{B_N} - \tau^2_{C_N}}\right]. \tag{11}$$

To obtain MSE for case-II, all the notations will remain same except, $E(\eta_{y_2}\eta_{x_1}) = 0 = E(\eta_{x_1}\eta_{x_2})$. Hence the MSE of $\bar{y}'_{N_i}$ for case-II as given below

$$MSE(\bar{y}'_{N_i})_{II} = \bar{Y}^2 + k_1^2\psi_{A_N} + k_2^2\psi_{B_N} + 2k_1k_2\psi_{C_N} - 2k_1\psi_{D_N}, \tag{12}$$

where

$$\begin{aligned}
\psi_{A_N} &= \left[\bar{Y}^2(1+\gamma_2 C_y^2) + b^2_{(i)}\bar{X}^2(\gamma_1+\gamma_2)C_x^2 - 2b_{(i)}\bar{X}\bar{Y}\gamma_2 C_{yx}\right],\\
\psi_{B_N} &= \bar{X}^2(\gamma_2 + \gamma_1)C_x^2,\\
\psi_{C_N} &= \bar{X}[b_{(i)}\bar{X}(\gamma_1+\gamma_2)C_x^2 - \bar{Y}\gamma_2 C_{yx}],\\
\psi_{D_N} &= \bar{Y}^2.
\end{aligned}$$

Which is minimum for

$$k_1^{opt} = \left[\frac{\psi_{B_N}\psi_{D_N}}{\psi_{A_N}\psi_{B_N} - \psi^2_{C_N}}\right],$$

and

$$k_2^{opt} = \left[-\frac{\psi_{C_N}\psi_{D_N}}{\psi_{A_N}\psi_{B_N} - \psi^2_{C_N}}\right].$$

$$MSE(\bar{y}^{'}_{N_i})_{II} = \left[\bar{Y}^2 - \frac{\psi_{B_N}\psi^2_{D_N}}{\psi_{A_N}\psi_{B_N} - \psi^2_{C_N}}\right]. \quad (13)$$

3 Results and Discussion

3.1 Numerical illustration

For the assessment of the proposed and competitor estimators, we consider a real life application in the form of Population-1 and an artificial population as Population-2.

3.1.1 Real life application

In this sub-section, we utilized the data set available in Singh (2003). This data set is recently utilized by Ali et al. (2019) for sensitivity issue by adding scramble response in it. Here, we are considering this data in absence of sensitivity. As there is a non-negative correlation exist between the study and auxiliary variates, also figures 1 and 2 show non-normality and presence of outliers, respectively, hence suitable for utilization of robust-regression tools. Results of percentage relative efficiency (PRE) are provided in Table 3. Some major characteristics of the population are as given below
X = Amount of non-real estate farm loans during 1977 and
Y = Amount of real estate farm loans during 1977.

$N = 50$	$\bar{Y} = 555.4345$	$\bar{X} = 878.1624$	$b_{(lts)} = 0.3484253$
$n = 20$	$S_y = 584.826$	$S_x = 1084.678$	$b_{(hbm)} = 0.4123359$
$\rho = 0.804$	$\beta_2(x) = 4.617048$	$b_{(OLS)} = 0.4334034$	$b_{(hpm)} = 0.4267937$
$n_1 = 16$	$C_x = 1.235168$	$b_{(lad)} = 0.3937749$	$b_{(tkz)} = 0.4187815$
$n_2 = 20$	$C_y = 1.052916$	$b_{(lms)} = 0.3396594$	$b_{(hmm)} = 0.3480814$

3.1.2 Simulation study

In this subsection, there is an assessment of proposed and existing estimators performed with the assumption that all the population parameters are known. But in numerous genuine circumstances, these parameters are mostly obscure and can't be speculated based on past information. Subsequently they should be evaluated. In such circumstances, an additional variability is presented in the evaluations that could invalid the hypothetical examinations. So in this sub-section, we are paying our attention regarding

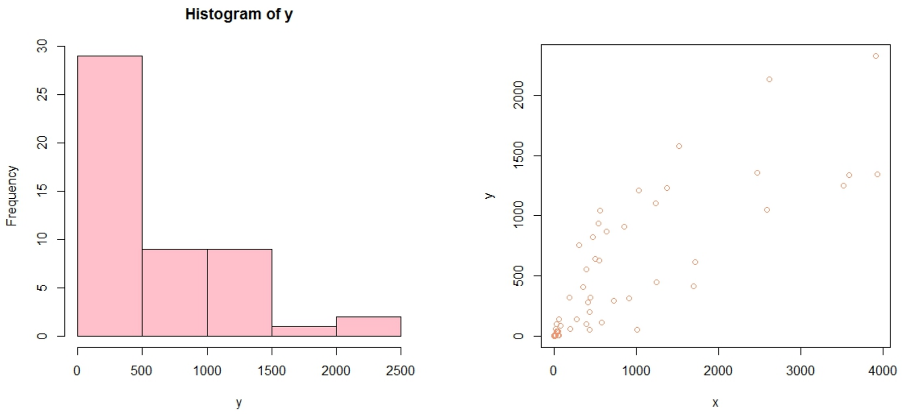

Figure 1: The graphical behavior of Population-1 through: **(a)** Historigram, **(b)** Scatter Plot

the PRE examinations at the point when obscure population parameters are assessed from the selected sample. For this purpose we are performing Monte Carlo simulation.

The simulation design is organized as follows: a random variable $X_i \sim G(2.6, 3.8)$ and random variable Y_i, which is presented as: $Y_i = h + RX_i + \varepsilon X_i^p$. Here we choose, $p = 1.6,\ h = 5,\ R = 2$ and $\varepsilon \sim N(0, 1)$ with $N = 1000$ (population size). Here, the simple random sampling (SRS) is considered for $n = 200$ and replicated 1000 times. We examine empirical MSEs' of $\bar{y}_{zb_i}, \bar{y}_{reg}$ and $\bar{y}_{N_i}$ as $MSE = \frac{\sum_{i=1}^{K'}(z_i - \bar{Z})^2}{K'}$. Using the results of empirical MSE we calculated PRE of each estimator, available in Table 4.

We consider same simulation design for two stage sampling. A sample size, $n_1 = 200$ is selected from (X_i, Y_i) at first stage and sample of size $n_2 = 160$ is selected at second stage. The Second stage sample is selected differently for case-I and case-II as per requirement of no-independence and independence with respect to initial stage sample i.e. n_1, respectively. The pattern of Figures 3 and 4 clearly show for the applicability of robust-regression tools. The results of PRE for case-I and case-II are provided in Table 4, where PRE is computed with respect to $V(\bar{y})$ as:

$$\text{PRE}(\hat{\theta}) = \frac{\text{Var}(\bar{y})}{\text{MSE}(\hat{\theta})} \times 100.$$

3.2 Discussion

Our findings which are based on the results of numerical illustration (Table 3 and 4) are highlighted as given below:

- $\bar{y}_{zb_9}$ is given maximum PRE as compared to all the competitors ratio-type-estimators, under SRS and two stage sampling schemes.

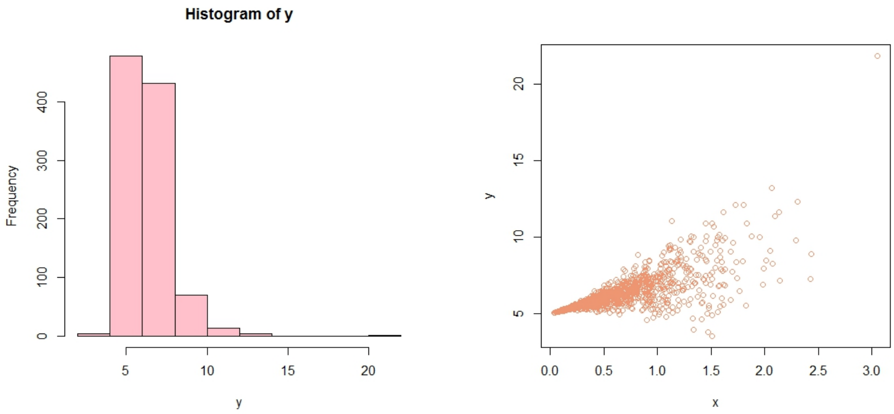

Figure 2: The graphical behavior of Population-2 through: **(a)** Historigram, **(b)** Scatter Plot

- $\bar{y}_{reg}$ is given maximum PRE as compared to all the competitors ratio-type-estimators, under SRS and first case of two stage sampling schemes. However, $\bar{y}_{zb_9}$ is performing better than the usual regression estimator in case-II of two stage sampling scheme.
- By ignoring fractional values in proposed class, we observe that all the members of proposed class are equally important under SRS and first case of two stage sampling schemes. However for case-II, $\bar{y}'_{N_2}$ is performing outclass among all other proposed estimators.
- All the proposed estimators have maximum PRE over sample mean estimator, $\bar{y}_{zb_i}$, $\bar{y}_{z_i}$ and $\bar{y}_{reg}$ under SRS and two stage sampling schemes.

According to the real life application and simulated results, we observed that, the proposed techniques outperform over existing and adapting ones.

4 Conclusion

In this study, we proposed a new family of robust-regression type estimators for mean estimation, under simple and two-stage random sampling schemes with quantitative supplementary information is available. We also derived the expressions of MSE and minimum MSE for the proposed family of estimators. A comparative study is conduced between new and some existing ones based on theoretical and empirical PRE results. The findings are clearly showed that the proposed class performs better as compared to the traditional regressions estimators, such as Zaman and Bulut (2019a) and Zaman (2019) estimators. Hence, it can be recommended by based on its performance to utilize them in real life applications. In future studies, we hope to extend this work for handling the sensitive issue, in light of Ali et al. (2019) article.

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Table 1: Reviewed robust ratio type estimators

Estimators	$b_{(i)}$	a	b
$\bar{y}_{zb_1}$	$b_{(lad)}$	1	0
$\bar{y}_{zb_2}$	$b_{(lad)}$	1	C_x
$\bar{y}_{zb_3}$	$b_{(lad)}$	1	$\beta_2(x)$
$\bar{y}_{zb_4}$	$b_{(lad)}$	$\beta_2(x)$	C_x
$\bar{y}_{zb_5}$	$b_{(lad)}$	C_x	$\beta_2(x)$
$\bar{y}_{zb_6}$	$b_{(lms)}$	1	0
$\bar{y}_{zb_7}$	$b_{(lms)}$	1	C_x
$\bar{y}_{zb_8}$	$b_{(lms)}$	1	$\beta_2(x)$
$\bar{y}_{zb_9}$	$b_{(lms)}$	$\beta_2(x)$	C_x
$\bar{y}_{zb_{10}}$	$b_{(lms)}$	C_x	$\beta_2(x)$
$\bar{y}_{zb_{11}}$	$b_{(lts)}$	1	0
$\bar{y}_{zb_{12}}$	$b_{(lts)}$	1	C_x
$\bar{y}_{zb_{13}}$	$b_{(lts)}$	1	$\beta_2(x)$
$\bar{y}_{zb_{14}}$	$b_{(lts)}$	$\beta_2(x)$	C_x
$\bar{y}_{zb_{15}}$	$b_{(lts)}$	C_x	$\beta_2(x)$
$\bar{y}_{zb_{16}}$	$b_{(hbm)}$	1	0
$\bar{y}_{zb_{17}}$	$b_{(hbm)}$	1	C_x
$\bar{y}_{zb_{18}}$	$b_{(hbm)}$	1	$\beta_2(x)$
$\bar{y}_{zb_{19}}$	$b_{(hbm)}$	$\beta_2(x)$	C_x
$\bar{y}_{zb_{20}}$	$b_{(hbm)}$	C_x	$\beta_2(x)$
$\bar{y}_{zb_{21}}$	$b_{(hpm)}$	1	0
$\bar{y}_{zb_{22}}$	$b_{(hpm)}$	1	C_x
$\bar{y}_{zb_{23}}$	$b_{(hpm)}$	1	$\beta_2(x)$
$\bar{y}_{zb_{24}}$	$b_{(hpm)}$	$\beta_2(x)$	C_x
$\bar{y}_{zb_{25}}$	$b_{(hpm)}$	C_x	$\beta_2(x)$
$\bar{y}_{zb_{26}}$	$b_{(tkz)}$	1	0
$\bar{y}_{zb_{27}}$	$b_{(tkz)}$	1	C_x
$\bar{y}_{zb_{28}}$	$b_{(tkz)}$	1	$\beta_2(x)$
$\bar{y}_{zb_{29}}$	$b_{(tkz)}$	$\beta_2(x)$	C_x
$\bar{y}_{zb_{30}}$	$b_{(tkz)}$	C_x	$\beta_2(x)$
$\bar{y}_{zb_{31}}$	$b_{(hmm)}$	1	0
$\bar{y}_{zb_{32}}$	$b_{(hmm)}$	1	C_x
$\bar{y}_{zb_{33}}$	$b_{(hmm)}$	1	$\beta_2(x)$
$\bar{y}_{zb_{34}}$	$b_{(hmm)}$	$\beta_2(x)$	C_x
$\bar{y}_{zb_{35}}$	$b_{(hmm)}$	C_x	$\beta_2(x)$

Table 2: Family members of proposed class

SRS Estimators	$b_{(i)}$	Two stage sampling Estimators
$\bar{y}_{N_1}$	$b_{(lad)}$	$\bar{y}'_{N_1}$
$\bar{y}_{N_2}$	$b_{(lms)}$	$\bar{y}'_{N_2}$
$\bar{y}_{N_3}$	$b_{(lts)}$	$\bar{y}'_{N_3}$
$\bar{y}_{N_4}$	$b_{(hbm)}$	$\bar{y}'_{N_4}$
$\bar{y}_{N_5}$	$b_{(hpm)}$	$\bar{y}'_{N_5}$
$\bar{y}_{N_6}$	$b_{(tky)}$	$\bar{y}'_{N_6}$
$\bar{y}_{N_7}$	$b_{(hmm)}$	$\bar{y}'_{N_7}$

Table 3: PRE of Population-1

	SRS				Two Stage Case-I				Two Stage Case-II		
$\hat{\theta}$	PRE	$\hat{\theta}$	PRE	$\hat{\theta}$	PRE	$\hat{\theta}$	PRE	$\hat{\theta}$	PRE	$\hat{\theta}$	PRE
$\bar{y}$	100	$\bar{y}_{zb22}$	1963.60	$\bar{y}'$	100	$\bar{y}'_{zb22}$	1952.38	$\bar{y}'$	100	$\bar{y}'_{zb22}$	532.62
$\bar{y}_{zb1}$	2132.71	$\bar{y}_{zb23}$	1975.68	$\bar{y}'_{zb1}$	2018.69	$\bar{y}'_{zb23}$	1957.34	$\bar{y}'_{zb1}$	571.04	$\bar{y}'_{zb23}$	535.38
$\bar{y}_{zb2}$	2137.66	$\bar{y}_{zb24}$	8084.92	$\bar{y}'_{zb2}$	2020.54	$\bar{y}'_{zb24}$	2844.65	$\bar{y}'_{zb2}$	572.16	$\bar{y}'_{zb24}$	1987.31
$\bar{y}_{zb3}$	2151.23	$\bar{y}_{zb25}$	1972.54	$\bar{y}'_{zb3}$	2025.57	$\bar{y}'_{zb25}$	1956.05	$\bar{y}'_{zb3}$	575.22	$\bar{y}'_{zb25}$	534.66
$\bar{y}_{zb4}$	8625.92	$\bar{y}_{zb26}$	1999.47	$\bar{y}'_{zb4}$	2871.05	$\bar{y}'_{zb26}$	1967.01	$\bar{y}'_{zb4}$	2198.24	$\bar{y}'_{zb26}$	540.82
$\bar{y}_{zb5}$	2147.70	$\bar{y}_{zb27}$	2004.01	$\bar{y}'_{zb5}$	2024.26	$\bar{y}'_{zb27}$	1968.82	$\bar{y}'_{zb5}$	574.42	$\bar{y}'_{zb27}$	541.85
$\bar{y}_{zb6}$	2464.93	$\bar{y}_{zb28}$	2016.93	$\bar{y}'_{zb6}$	2132.02	$\bar{y}'_{zb28}$	1973.80	$\bar{y}'_{zb6}$	645.14	$\bar{y}'_{zb28}$	544.68
$\bar{y}_{zb7}$	2470.94	$\bar{y}_{zb29}$	8223.43	$\bar{y}'_{zb7}$	2133.89	$\bar{y}'_{zb29}$	2851.71	$\bar{y}'_{zb7}$	646.47	$\bar{y}'_{zb29}$	2036.75
$\bar{y}_{zb8}$	2487.41	$\bar{y}_{zb30}$	2013.20	$\bar{y}'_{zb8}$	2138.99	$\bar{y}'_{zb30}$	1972.51	$\bar{y}'_{zb8}$	650.10	$\bar{y}'_{zb30}$	543.95
$\bar{y}_{zb9}$	9252.26	$\bar{y}_{zb31}$	2408.89	$\bar{y}'_{zb9}$	2898.26	$\bar{y}'_{zb31}$	2114.29	$\bar{y}'_{zb9}$	2580.40	$\bar{y}'_{zb31}$	632.76
$\bar{y}_{zb10}$	2483.13	$\bar{y}_{zb32}$	2414.72	$\bar{y}'_{zb10}$	2137.67	$\bar{y}'_{zb32}$	2116.16	$\bar{y}'_{zb10}$	649.16	$\bar{y}'_{zb32}$	634.05
$\bar{y}_{zb11}$	2406.64	$\bar{y}_{zb33}$	2430.70	$\bar{y}'_{zb11}$	2113.57	$\bar{y}'_{zb33}$	2121.25	$\bar{y}'_{zb11}$	632.26	$\bar{y}'_{zb33}$	637.58
$\bar{y}_{zb12}$	2412.46	$\bar{y}_{zb34}$	9182.23	$\bar{y}'_{zb12}$	2115.44	$\bar{y}'_{zb34}$	2895.38	$\bar{y}'_{zb12}$	633.55	$\bar{y}'_{zb34}$	2518.50
$\bar{y}_{zb13}$	2428.42	$\bar{y}_{zb35}$	2426.54	$\bar{y}'_{zb13}$	2120.53	$\bar{y}'_{zb35}$	2119.93	$\bar{y}'_{zb13}$	637.08	$\bar{y}'_{zb35}$	636.66
$\bar{y}_{zb14}$	9179.14	$\bar{y}_{reg}$	9420.16	$\bar{y}'_{zb14}$	2895.25	$\bar{y}'_{reg}$	2905.02	$\bar{y}'_{zb14}$	2515.99	$\bar{y}'_{reg}$	1977.18
$\bar{y}_{zb15}$	2424.27	$\bar{y}_{N1}$	9531.63	$\bar{y}'_{zb15}$	2119.21	$\bar{y}'_{N1}$	3015.69	$\bar{y}'_{zb15}$	636.16	$\bar{y}'_{N1}$	18565.24
$\bar{y}_{zb16}$	2032.70	$\bar{y}_{N2}$	9531.26	$\bar{y}'_{zb16}$	1980.27	$\bar{y}'_{N2}$	3015.93	$\bar{y}'_{zb16}$	548.39	$\bar{y}'_{N2}$	26053.66
$\bar{y}_{zb17}$	3237.34	$\bar{y}_{N3}$	9531.13	$\bar{y}'_{zb17}$	1982.10	$\bar{y}'_{N3}$	3015.53	$\bar{y}'_{zb17}$	549.44	$\bar{y}'_{N3}$	25037.30
$\bar{y}_{zb18}$	2050.05	$\bar{y}_{N4}$	9531.07	$\bar{y}'_{zb18}$	1987.09	$\bar{y}'_{N4}$	3015.48	$\bar{y}'_{zb18}$	552.33	$\bar{y}'_{N4}$	16030.02
$\bar{y}_{zb19}$	8332.40	$\bar{y}_{N5}$	9531.91	$\bar{y}'_{zb19}$	2857.09	$\bar{y}'_{N5}$	3015.02	$\bar{y}'_{zb19}$	2077.35	$\bar{y}'_{N5}$	14256.48
$\bar{y}_{zb20}$	2046.75	$\bar{y}_{N6}$	9531.55	$\bar{y}'_{zb20}$	1985.80	$\bar{y}'_{N6}$	3015.41	$\bar{y}'_{zb20}$	551.58	$\bar{y}'_{N6}$	15215.86
$\bar{y}_{zb21}$	1959.18	$\bar{y}_{N7}$	9531.64	$\bar{y}'_{zb21}$	1950.56	$\bar{y}'_{N7}$	3015.25	$\bar{y}'_{zb21}$	531.61	$\bar{y}'_{N7}$	25080.03

Table 4: PRE of Population-2

	SRS				Two Stage Case-I				Two Stage Case-II		
$\hat{\theta}$	PRE	$\hat{\theta}$	PRE	$\hat{\theta}$	PRE	$\hat{\theta}$	PRE	$\hat{\theta}$	PRE	$\hat{\theta}$	PRE
$\bar{y}$	100	$\bar{y}_{zb22}$	7502.35	$\bar{y}'$	100	$\bar{y}'_{zb22}$	11903.28	$\bar{y}'$	100	$\bar{y}'_{zb22}$	3050.51
$\bar{y}_{zb1}$	2276.96	$\bar{y}_{zb23}$	34896.04	$\bar{y}'_{zb1}$	5992.13	$\bar{y}'_{zb23}$	19865.52	$\bar{y}'_{zb1}$	998.62	$\bar{y}'_{zb23}$	15238.09
$\bar{y}_{zb2}$	7237.82	$\bar{y}_{zb24}$	30406.97	$\bar{y}'_{zb2}$	12058.50	$\bar{y}'_{zb24}$	18904.15	$\bar{y}'_{zb2}$	3026.99	$\bar{y}'_{zb24}$	12030.34
$\bar{y}_{zb3}$	33694.82	$\bar{y}_{zb25}$	38159.11	$\bar{y}'_{zb3}$	19799.42	$\bar{y}'_{zb25}$	20230.62	$\bar{y}'_{zb3}$	15053.22	$\bar{y}'_{zb25}$	18465.89
$\bar{y}_{zb4}$	29157.60	$\bar{y}_{zb26}$	2249.37	$\bar{y}'_{zb4}$	19045.03	$\bar{y}'_{zb26}$	5905.88	$\bar{y}'_{zb4}$	11884.51	$\bar{y}'_{zb26}$	1028.33
$\bar{y}_{zb5}$	37168.84	$\bar{y}_{zb27}$	7092.51	$\bar{y}'_{zb5}$	20314.48	$\bar{y}'_{zb27}$	11868.73	$\bar{y}'_{zb5}$	18254.59	$\bar{y}'_{zb27}$	3179.62
$\bar{y}_{zb6}$	2247.27	$\bar{y}_{zb28}$	32992.97	$\bar{y}'_{zb6}$	5921.10	$\bar{y}'_{zb28}$	19659.31	$\bar{y}'_{zb6}$	1047.90	$\bar{y}'_{zb28}$	16249.63
$\bar{y}_{zb7}$	7081.52	$\bar{y}_{zb29}$	28450.23	$\bar{y}'_{zb7}$	11902.29	$\bar{y}'_{zb29}$	18872.06	$\bar{y}'_{zb7}$	3282.46	$\bar{y}'_{zb29}$	12836.31
$\bar{y}_{zb8}$	32938.70	$\bar{y}_{zb30}$	36561.93	$\bar{y}'_{zb8}$	19684.77	$\bar{y}'_{zb30}$	20211.01	$\bar{y}'_{zb8}$	17047.55	$\bar{y}'_{zb30}$	19601.05
$\bar{y}_{zb9}$	38160.15	$\bar{y}_{zb31}$	2255.60	$\bar{y}'_{zb9}$	20316.48	$\bar{y}'_{zb31}$	5907.31	$\bar{y}'_{zb9}$	20472.14	$\bar{y}'_{zb31}$	1026.92
$\bar{y}_{zb10}$	36514.21	$\bar{y}_{zb32}$	7125.18	$\bar{y}'_{zb10}$	20230.06	$\bar{y}'_{zb32}$	11871.89	$\bar{y}'_{zb10}$	20469.01	$\bar{y}'_{zb32}$	3172.31
$\bar{y}_{zb11}$	2227.07	$\bar{y}_{zb33}$	33153.25	$\bar{y}'_{zb11}$	5932.43	$\bar{y}'_{zb33}$	19661.72	$\bar{y}'_{zb11}$	1027.16	$\bar{y}'_{zb33}$	16192.56
$\bar{y}_{zb12}$	6976.16	$\bar{y}_{zb34}$	28610.46	$\bar{y}'_{zb12}$	11927.26	$\bar{y}'_{zb34}$	18875.01	$\bar{y}'_{zb12}$	3173.54	$\bar{y}'_{zb34}$	12790.44
$\bar{y}_{zb13}$	32410.50	$\bar{y}_{zb35}$	36702.23	$\bar{y}'_{zb13}$	19703.53	$\bar{y}'_{zb35}$	20212.81	$\bar{y}'_{zb13}$	16202.11	$\bar{y}'_{zb35}$	19538.01
$\bar{y}_{zb14}$	27874.02	$\bar{y}_{reg}$	41210.94	$\bar{y}'_{zb14}$	18926.26	$\bar{y}'_{reg}$	20794.12	$\bar{y}'_{zb14}$	12798.11	$\bar{y}'_{reg}$	16919.97
$\bar{y}_{zb15}$	36044.12	$\bar{y}_{N1}$	41236.83	$\bar{y}'_{zb15}$	20244.03	$\bar{y}'_{N1}$	20808.04	$\bar{y}'_{zb15}$	19548.56	$\bar{y}'_{N1}$	55716.02
$\bar{y}_{zb16}$	2299.48	$\bar{y}_{N2}$	41236.26	$\bar{y}'_{zb16}$	5978.75	$\bar{y}'_{N2}$	20808.47	$\bar{y}'_{zb16}$	987.61	$\bar{y}'_{N2}$	60743.40
$\bar{y}_{zb17}$	7357.54	$\bar{y}_{N3}$	41236.53	$\bar{y}'_{zb17}$	12029.12	$\bar{y}'_{N3}$	20808.11	$\bar{y}'_{zb17}$	2971.77	$\bar{y}'_{N3}$	59049.46
$\bar{y}_{zb18}$	34250.98	$\bar{y}_{N4}$	41236.71	$\bar{y}'_{zb18}$	19778.35	$\bar{y}'_{N4}$	20808.28	$\bar{y}'_{zb18}$	14619.03	$\bar{y}'_{N4}$	54175.60
$\bar{y}_{zb19}$	29729.47	$\bar{y}_{N5}$	41236.16	$\bar{y}'_{zb19}$	19018.77	$\bar{y}'_{N5}$	20808.76	$\bar{y}'_{zb19}$	11543.59	$\bar{y}'_{N5}$	56327.54
$\bar{y}_{zb20}$	37635.41	$\bar{y}_{N6}$	41236.03	$\bar{y}'_{zb20}$	20299.14	$\bar{y}'_{N6}$	20808.22	$\bar{y}'_{zb20}$	17753.99	$\bar{y}'_{N6}$	59162.35
$\bar{y}_{zb21}$	2326.47	$\bar{y}_{N7}$	41236.55	$\bar{y}'_{zb21}$	5921.55	$\bar{y}'_{N7}$	20808.41	$\bar{y}'_{zb21}$	1003.25	$\bar{y}'_{N7}$	59026.51