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A note on weakly s-normal subgroups of finite groups

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Abstract. In this paper, we investigate the influence of the certain subgroups of fixed prime power order on the p-supersolubility of finite groups. Many recent results are extended.

Keywords: p-soluble, p-supersoluble, weakly s-normal

MSC 2020 classification: primary 20D10, secondary 20D20

1 Introduction

Throughout this paper, all groups are assumed to be finite. The terminology and notions employed agree with standard usage, as in Doerk and Hawkes [5]. G always denotes a finite group, p denotes a prime and $Z_{\mathfrak{U}}(G)$ is the $\mathfrak{U}$-hypercenter of G, i.e., the product of all normal subgroups H of G such that all G-chief factors of H are cyclic. The generalized Fitting subgroup $F^*(G)$ of G is the unique maximal normal quasinilpotent subgroup of G (see [8, X, 13]). We use $F_p(G)$ to denote the p-Fitting subgroup of G. The generalized p-Fitting subgroup $F_p^*(G)$ is defined to be as the normal subgroup of G such that $F^*(G/O_{p'}(G)) = F_p^*(G)/O_{p'}(G)$ (see [3]).

A subgroup H of G is said to be s-permutable [9] in G if H permutes with every Sylow subgroup of G; A subgroup H of G is called weakly s-permutable [15] in G if there is a subnormal subgroup T of G such that $G = HT$ and $H \cap T \leq H_{sG}$, where H_{sG} is the subgroup of H generated by all those subgroups of H which are s-permutable in G.

A subgroup H of G is said to be s-semipermutable [4] in G if H permutes with every Sylow p-subgroup G_p of G with $(|H|, p) = 1$; A subgroup H of G is called weakly s-semipermutable [11] in G if there are a subnormal subgroup T of G and an s-semipermutable subgroup H_{ssG} of G contained in H such that $G = HT$ and $H \cap T \leq H_{ssG}$.

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A subgroup H of G is said to be s-permutably embedded [1] in G if for each prime p dividing $|H|$, a Sylow p-subgroup of H is also a Sylow p-subgroup of some s-permutable subgroup of G; A subgroup H of G is called weakly s-permutably embedded [10] in G if there are a subnormal subgroup T of G and an s-permutably embedded subgroup H_{seG} of G contained in H such that $G = HT$ and $H \cap T \leq H_{seG}$.

In [12], Li and Qiao introduced the following concept which covers the above mentioned subgroups:

A subgroup H of G is called weakly s-normal in G if there are a subnormal subgroup T of G and a subgroup H_* of H such that $G = HT$ and $H \cap T \leq H_*$, where H_* is a subgroup of H which is either s-permutably embedded or s-semipermutable in G.

In [12], the authors obtained some results on p-nilpotency and supersolubility of finite groups by using the notion of weakly s-normal subgroup. In this paper, we continue to investigate this concept and arrive at the following main result about p-supersolubility of finite groups.

Theorem 1. *Let E and X be p-soluble normal subgroups of G such that $F_p(E) \leq X \leq E$, where p is a prime divisor of $|E|$. Suppose that G/E is p-supersoluble, and a Sylow p-subgroup P of X has a subgroup D with $1 < |D| < |P|$ such that every subgroup H of P with order $|D|$ and every cyclic subgroup of P with order 4 (if P is a nonabelian 2-group and $|D| = 2$) is weakly s-normal in G. Then G is p-supersoluble.*

An application of Theorem 1 not only unifies many recent results in the literature, but also gives a new proof.

2 Preliminaries

Lemma 1 ([12, Lemma 2.5])**.** *Let U be a weakly s-normal subgroup of G and N a normal subgroup of G. Then*

(1) *If $U \leq H \leq G$, then U is weakly s-normal in H.*

(2) *Suppose that U is a p-group for some prime p. If $N \leq U$, then U/N is weakly s-normal in G/N.*

(3) *Suppose that U is a p-group for some prime p and N is a p'-subgroup. Then UN/N is weakly s-normal in G/N.*

(4) *Suppose that U is a p-group for some prime p and U is neither s-semipermutable nor s-permutably embedded in G. Then G has a normal subgroup M such that $|G : M| = p$ and $G = MU$.*

(5) *If $U \leq O_p(G)$ for some prime p, then U is weakly s-permutable in G.*

Lemma 2. *Let P be a normal p-subgroup of G. If there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with order $|D|$ and every cyclic subgroup of P with order 4 (if P is a nonabelian 2-group and $|D| = 2$) is weakly s-normal in G, then $P \leq Z_{\mathfrak{U}}(G)$.*

Proof. This follows from Lemma 1(5) and [17, Theorem]. QED

Lemma 3 ([3, Lemma 2.10])**.** *Let p be a prime and G a group.*
(1) *$Soc(G) \leq F_p^*(G)$.*
(2) *$O_{p'}(G) \leq F_p^*(G)$.*
In fact, $F^*(G/O_{p'}(G)) = F_p^*(G/O_{p'}(G)) = F_p^*(G)/O_{p'}(G)$.
(3) *If $F_p^*(G)$ is p-soluble, then $F_p^*(G) = F_p(G)$.*

Lemma 4 ([16, Theorem C])**.** *Let E be a normal subgroup of G. If every G-chief factor of $F^*(E)$ is cyclic, then every G-chief factor of E is also cyclic.*

Lemma 5 ([11, Theorem 3.3])**.** *Let P be a Sylow p-subgroup of a group G, where p is a prime dividing $|G|$. Suppose that there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with order $|D|$ and every cyclic subgroup of P with order 4 (if P is a nonabelian 2-group and $|D| = 2$) is s-semipermutable in G. Then G is p-supersoluble.*

Combining Lemma 1(5) and [15, Lemma 2.11], we have the following lemma.

Lemma 6. *Let N be an elementary abelian normal p-subgroup of a group G. If there is a subgroup D of N with $1 < |D| < |N|$ such that every subgroup of N with order $|D|$ is weakly s-normal in G, then there exists a maximal subgroup M of N such that M is normal in G.*

Lemma 7 ([13, Lemma 2.3])**.** *Suppose that H is s-permutable in G, and let P be a Sylow p-subgroup of H. If $H_G = 1$, then P is s-permutable in G.*

Lemma 8 ([14, Lemma A])**.** *If P is an s-permutable p-subgroup of a group G for some prime p, then $N_G(P) \geq O^p(G)$.*

Lemma 9 ([2, Theorem 2.1.6])**.** *If G is p-supersoluble and $O_{p'}(G) = 1$, then the Sylow p-subgroup of G is normal in G.*

The following Lemma is a coronary of [12, Theorem 3.2].

Lemma 10. *Let P be a Sylow p-subgroup of a group G, where p is the smallest prime dividing $|G|$. Suppose that there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with order $|D|$ and every cyclic subgroup of P with order 4 (if P is a nonabelian 2-group and $|D| = 2$) is weakly s-normal in G. Then G is p-nilpotent.*

3 Main Results

Theorem 2. *Let P be a Sylow p-subgroup of a p-soluble group G, where p is a prime divisor of $|G|$. If every cyclic subgroup of P with order p and 4 (if P is a nonabelian 2-group) is weakly s-normal in G, then G is p-supersoluble.*

Proof. Suppose that the theorem is false and let G be a counterexample of minimal order. Assume that $O_{p'}(G) \neq 1$. From Lemma 1(3) it is easy to see that every cyclic subgroup of $P/O_{p'}(G)$ with order p and 4 is weakly s-normal in $G/O_{p'}(G)$. The minimal choice of G yields that $G/O_{p'}(G)$ is p-supersoluble and so G is also p-supersoluble. This contradiction implies that $O_{p'}(G) = 1$. Since G is p-soluble, we have $O_p(G) \neq 1$. In view of Lemma 3, $F^*(G) = F_p^*(G) = F_p(G) = O_p(G)$. By hypothesis every cyclic subgroup of $F^*(G)$ with order p and 4 is weakly s-normal in G. By Lemma 2, $F^*(G) \leq Z_{\mathfrak{U}}(G)$. Applying Lemma 4, G is p-supersoluble. QED

Theorem 3. *Let P be a Sylow p-subgroup of a p-soluble group G, where p is a prime divisor of $|G|$. If every maximal subgroup of P is weakly s-normal in G, then G is p-supersoluble.*

Proof. Suppose that the theorem is false and G is a counterexample with minimal order.

Assume that $O_{p'}(G) \neq 1$. We consider the factor group $G/O_{p'}(G)$. It is easy to see that every maximal subgroup of $PO_{p'}(G)/O_{p'}(G)$ is weakly s-normal in $G/O_{p'}(G)$ by Lemma 1(3). Therefore, $G/O_{p'}(G)$ satisfies the hypothesis of our theorem. The minimal choice of G implies that $G/O_{p'}(G)$ is p-supersoluble and so G is p-supersoluble, a contradiction. Hence $O_{p'}(G) = 1$. Let N be a minimal normal group of G. Obviously, $N \leq O_p(G)$. It is easy to see that G/N satisfies the hypothesis of the theorem. Hence the minimal choice of G yields that G/N is p-supersoluble. Since the class of all p-supersoluble groups is a saturated formation, it follows that N is the unique minimal normal subgroup of G and $\Phi(G) = 1$. Consequently, G has a maximal subgroup M such that $G = MN$. Clearly, $P = P \cap NM = N(P \cap M)$. Since $P \cap M < P$, we may take a maximal subgroup P_1 of P such that $P \cap M \leq P_1$. Then $P = NP_1$ and $N \nleq P_1$. By hypothesis, P_1 is weakly s-normal in G. Then there are a subnormal subgroup T of G and a subgroup $(P_1)_*$ of P_1 such that $G = P_1T$ and $P_1 \cap T \leq (P_1)_*$, where $(P_1)_*$ is a subgroup of P_1 which is either s-permutably embedded or s-semipermutable in G. Since $|G : T|$ is a power of p, we have $N \leq O^p(G) \leq T$. It follows that $P_1 \cap N = (P_1)_* \cap N$. Next we prove $(P_1)_*$ is s-semipermutable in G. Assume that $(P_1)_*$ is s-permutably embedded in G. Then there is an s-permutable subgroup K of G such that $(P_1)_*$ is a Sylow p-subgroup of K. If $K_G \neq 1$, then $N \leq K_G \leq K$ since N is the unique minimal normal subgroup of

G. It follows that $N \leq (P_1)_* \leq P_1$, a contradiction. If $K_G = 1$, then, by Lemma 5, we have $(P_1)_*$ is s-permutable in G and so $(P_1)_*$ is s-semipermutable in G. Now we have $(P_1)_*Q = Q(P_1)_*$ for any Sylow q-subgroup Q of G, $q \neq p$. Then, there holds $[P_1 \cap N, Q] \leq N \cap (P_1)_*Q = N \cap (P_1)_* = N \cap P_1$. Obviously, $N \cap P_1$ is normalized by P. Therefore $N \cap P_1$ is normal in G. By the minimal normality of N we have $N \cap P_1 = 1$ or $N \cap P_1 = N$. If the latter holds, then $N \leq P_1$, a contradiction. Hence $N \cap P_1 = 1$. Then $|N : P_1 \cap N| = |NP_1 : P_1| = |P : P_1| = p$ and so $P_1 \cap N$ is a maximal of N. This shows tha $|N| = p$. It follows that G is p-supersoluble since G/N is p-supersoluble, a contradiction. QED

Theorem 4. *Let P be a Sylow p-subgroup of a p-soluble group G, where p is a prime divisor of $|G|$. If there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with order $|D|$ and every cyclic subgroup of P with order 4 (if P is a nonabelian 2-group and $|D| = 2$) is weakly s-normal in G, then G is p-supersoluble.*

Proof. Suppose that the theorem is false and let G be a counterexample of minimal order. We may assume that $p > 2$ from Lemma 10.

(1) $O_{p'}(G) = 1$.

Assume that $O_{p'}(G) \neq 1$. In view of Lemma 1(3), it is easy to see that $G/O_{p'}(G)$ªÑ satisfies the hypothesis of the theorem. Then, by the minimal choice of G, $G/O_{p'}(G)$ is p-supersoluble and so G is p-supersoluble, a contradiction.

(2) $|D| > p$ and $|P : D| > p$. In particular, $|P| \geqslant p^4$.

This follows from Theorems 2 and 3.

(3) If $H \leq P$ and $|H| = |D|$, then H is either s-permutably embedded or s-semipermutable in G.

By hypothesis, H is weakly s-normal in G. If H is neither s-permutably embedded nor s-semipermutable in G, then there exists a normal subgroup M of G such that $|G : M| = p$ by Lemma 1(4). By Step (2) and Lemma 1(1), it is easy to see that M satisfies the hypothesis of the theorem. The minimal choice of G implies that M is p-supersoluble. By Step (1) we have $O_{p'}(M) = 1$. Since $P \cap M$ is a Sylow p-subgroup of M, it follows from Lemma 9 that $P \cap M$ is normal in M. Obviously, $P \cap M$ is also normal in G. Applying Lemma 2, every G-chief factor of $P \cap M$ is cyclic. Hence every p-chief factors of G under M is cyclic. Since $|G/M| = p$, it follows that G is p-supersoluble. This contradiction shows that H is either s-permutably embedded or s-semipermutable in G.

(4) There is a subgroup R of P with order $|D|$ such that R is not s-semipermutable.

This follows from Lemma 5.

(5) If K is an s-permutable subgroup of G, then K is p-supersoluble.

If $KP < G$, then it is easy to see that KP satisfies the hypothesis of the theorem from Lemma 1(1). By the minimal choice of G we have KP is p-supersoluble. In particular, K is p-supersoluble. If $KP = G$, then G has a normal subgroup M of index p which contains K since K is subnormal in G. By Step (2) and Lemma 1(1), it is easy to see that M satisfies the hypothesis of the theorem. The minimal choice of G implies that M is p-supersoluble. Consequently, K is p-supersoluble.

(6) If K is an s-permutable subgroup of G, then the Sylow p-subgroup K_p of K is subnormal in G.

Since K is s-permutable in G, we have K is subnormal in G. Consequently, $O_{p'}(K)$ is subnormal in G. By Step (1), $O_{p'}(K) \leq O_{p'}(G) = 1$. By Step (5) K is p-supersoluble. It follows from Lemma 9 that K_p is normal in K and so K_p is subnormal in G.

(7) Final contradiction.

By Steps (3) and (4), R is s-permutably embedded in G. Then there is an s-permutable subgroup K of G such that R is a Sylow p-subgroup of K. By Step (6), we have tha R is subnormal in G. It follows that $R \leq O_p(G)$. By [13, Lemma 2.4], R is s-permutable in G. In particular, R is s-semipermutable in G, contrary to (4). QED

Theorem 5. *Let E be a p-soluble normal subgroup of G and P a Sylow p-subgroup of E, where p is a prime divisor of $|E|$. Suppose that there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with order $|D|$ and every cyclic subgroup of P with order 4 (if P is a nonabelian 2-group and $|D| = 2$) is weakly s-normal in G. Then $E/O_{p'}(E) \leq Z_{\mathfrak{U}}(G/O_{p'}(E))$.*

Proof. By Lemma 1(1), every subgroup H of P with order $|D|$ and every cyclic subgroup of P with order 4 (if P is a nonabelian 2-group and $|D| = 2$) is weakly s-normal in E. By Theorem 4 E is p-supersoluble. If $O_{p'}(E) \neq 1$, then from Lemma 1(3) the hypothesis is still true for $(G/O_{p'}(E), E/O_{p'}(E))$. By induction, $E/O_{p'}(E) = (E/O_{p'}(E))/O_{p'}(E/O_{p'}(E)) \leq Z_{\mathfrak{U}}((G/O_{p'}(E))/(O_{p'}(E/O_{p'}(E))))$ $= Z_{\mathfrak{U}}(G/O_{p'}(E))$. Now assume that $O_{p'}(E) = 1$. By virtue of Lemma 9, $P \trianglelefteq E$. Obviously, P is also normal in G. Since E is p-soluble, it follows from Lemma 3 that $F^*(E) = F_p^*(E) = F_p(E) = O_p(E) = P$. By Lemma 2, $F^*(E) \leq Z_{\mathfrak{U}}(G)$. Applying Lemma 4, $E \leq Z_{\mathfrak{U}}(G)$. QED

Theorem 6. *Let E and X be p-soluble normal subgroups of G such that $F_p(E) \leq X \leq E$, where p is a prime divisor of $|E|$. Suppose that a Sylow p-subgroup P of X has a subgroup D with $1 < |D| < |P|$ such that every subgroup H of P with order $|D|$ and every cyclic subgroup of P with order 4* *(if P is a nonabelian 2-group and $|D| = 2$) is weakly s-normal in G. Then $E/O_{p'}(E) \leq Z_{\mathfrak{U}}(G/O_{p'}(E))$.*

Proof. By Theorem 5, $X/O_{p'}(X) \leq Z_{\mathfrak{U}}(G/O_{p'}(X))$. Since $F_p(E) \leq X \leq E$, it is easy to see that $O_{p'}(X) = O_{p'}(E)$. Hence $X/O_{p'}(E) \leq Z_{\mathfrak{U}}(G/O_{p'}(E))$. Consequently, $F_p(E)/O_{p'}(E) \leq Z_{\mathfrak{U}}(G/O_{p'}(E))$. Since E is p-soluble, it follows from Lemma 3 that $F^*(E/O_{p'}(E)) = F_p^*(E/O_{p'}(E)) = F_p(E)/O_{p'}(E) \leq Z_{\mathfrak{U}}(G/O_{p'}(E))$. Applying Lemma 4, $E/O_{p'}(E) \leq Z_{\mathfrak{U}}(G/O_{p'}(E))$. QED

Proof of Theorem 1 By Theorem 6, $E/O_{p'}(E) \leq Z_{\mathfrak{U}}(G/O_{p'}(E))$. Since $(G/O_{p'}(E))/(E/O_{p'}(E)) \cong G/E$ is p-supersoluble, it follows that $G/O_{p'}(E)$ is p-supersoluble and so G is p-supersoluble.

4 Some Applications

Corollary 1. *Let p be an odd prime dividing $|G|$ and P a Sylow p-subgroup of G. Suppose that there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with order $|H| = |D|$ is weakly s-normal in G and $N_G(P)$ is p-nilpotent. Then G is p-nilpotent.*

Proof. Assume that the assertion is false and let G be a counterexample of minimal order. Then:

(1) If $P \leq U < G$, then U is p-nilpotent.

By Lemma 1(1), every subgroup H of P with order $|D|$ is weakly s-permutably embedded in U. Since $N_U(P) \leq N_G(P)$ and $N_G(P)$ is p-nilpotent, it follows that $N_U(P)$ is p-nilpotent. Hence U satisfies the hypothesis of the theorem and so U is p-nilpotent by the minimal choice of G.

(2) $O_p(G) \neq 1$.

Consider the group $Z(J(P))$, where $J(P)$ is the Thompson subgroup of P. If $N_G(Z(J(P))) < G$, then $N_G(Z(J(P))$ is p-nilpotent by Step (1). Then G is p-nilpotent by [6, Theorem 8.3.1], a contradiction. Hence $N_G(Z(J(P))) = G$ and $1 < Z(J(P)) \leq O_p(G) < P$.

(3) G is p-soluble.

Let

$$\overline{G} = G/O_p(G),\ \overline{P} = P/O_p(G),\ \overline{K} = Z(J(\overline{P})),\ G_1/O_p(G) = N_{\overline{G}}(Z(J(\overline{P}))).$$

Since $O_p(\overline{G}) = 1$, we have $N_{\overline{G}}(Z(J(\overline{P})) < \overline{G}$. Thus $G_1 < G$. By (1), we have G_1 is p-nilpotent. Then $N_{\overline{G}}(Z(J(\overline{P}))$ is p-nilpotent. Thus $\overline{G}$ is p-nilpotent by [6, Theorem 8.3.1]. Consequently, G is p-soluble.

(4) $O_{p'}(G) = 1$.

If $O_{p'}(G) \neq 1$, then $\overline{G}$ satisfies the hypothesis of the theorem by Lemma 1(3). The minimal choice of G implies that $G/O_{p'}(G)$ is p-nilpotent and so is G, a contradiction.

(5) Final contradiction.

Applying Theorem 1, G is p-supersoluble. In view of Lemma 9, P is normal in G. Therefore, $G = N_G(P)$ is p-nilpotent by hypothesis, a contradiction. QED

Corollary 2 ([22, Theorem 3.2])**.** *Let p be an odd prime dividing $|G|$ and P a Sylow p-subgroup of G. Suppose that there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with order $|H| = |D|$ is weakly s-permutably embedded in G and $N_G(P)$ is p-nilpotent. Then G is p-nilpotent.*

Corollary 3 ([20, Theorem 3.9])**.** *Let p be an odd prime dividing $|G|$ and P a Sylow p-subgroup of G. Suppose that there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with order $|D|$ is s-permutably embedded in G and $N_G(P)$ is p-nilpotent. Then G is p-nilpotent.*

Corollary 4 ([18, Theorem 3.4])**.** *Let p be an odd prime dividing $|G|$ and P a Sylow p-subgroup of G. Suppose that there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with order $|H| = |D|$ is weakly s-semipermutable in G and $N_G(P)$ is p-nilpotent. Then G is p-nilpotent.*

Corollary 5 ([7, Theorem 3.1])**.** *Let p be an odd prime dividing $|G|$ and P a Sylow p-subgroup of G. Suppose that there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with order $|H| = |D|$ is s-semipermutable in G and $N_G(P)$ is p-nilpotent. Then G is p-nilpotent.*

Corollary 6 ([21, Theorem 3.1])**.** *Let p be an odd prime dividing $|G|$ and P a Sylow p-subgroup of G. If $N_G(P)$ is p-nilpotent and there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with order $|D|$ is s-permutable in G, then G is p-nilpotent.*

Corollary 7 ([19, Theorem 3.1])**.** *Let p be an odd prime dividing $|G|$ and P a Sylow p-subgroup of G. Suppose that there exists a subgroup D of P with $1 < |D| < |P|$ such that every subgroup H of P with order $|H| = |D|$ is c^*-normal in G and $N_G(P)$ is p-nilpotent. Then G is p-nilpotent.*

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