decreases, the market share and the demand of new and more productive varieties made available increases as long as they are on the technological frontier.

5 Structural changes and the scale effect

One of the most striking characteristics of the moving equilibrium we have so far described is that it allows us to represent the effects of ongoing patent innovations which take place together with process innovations. Considering both kinds of innovations gives a more complete picture of the effects of R&D activities and it produces a setup in which the rate of growth of patent innovations varies across time according to workers' distribution between the final and the innovative sectors considered in the model.

In the period in which technology of type i is available, we know from expression (12) that the rate of innovation is proportional to the number of workers employed in the R&D sector, and this number L_R, derived from (33)-(34) when $\dot{V}_i = 0$, depends on the value of b_i, that is

$$L_R = \frac{Lb_i(1-\alpha) - a\rho\alpha}{(1-\alpha)b_i + \alpha} \tag{36}$$

As in Grossman and Helpman (1991), we assume that L is sufficiently large to allow patent innovations to take place: this requires that $L > a\rho\alpha/b_i(1-\alpha)$. Once more, it is readily verifiable that when $b_i = 1$ we obtain the same results as in Grossman and Helpman (1991).

Expression (36) shows that the number of workers employed in the innovative sector is an increasing function of b_i because

$$\frac{\partial L_R}{\partial b_i} = \frac{(1-\alpha)\alpha(L + a\rho)}{((1-\alpha)b_i + \alpha)^2} > 0$$

Therefore, when there are at least two different types of firms producing using different technologies, and the innovative sector intensifies its research in finding new patents for the production of new goods employing the more productive technologies, then any time a new patent is produced and implemented the value of b_i increases. As b_i increases, the final sector in aggregate becomes more productive and, therefore, more workers are made available to be employed in the R&D sector. Moreover, the growth rate of new varieties increases as it is shown by the following expression

$$g_i = g = \frac{L_R}{a} = \frac{Lb_i\,(1-\alpha)\,/a - \rho\alpha}{(1-\alpha)\,b_i + \alpha} \tag{37}$$

The growth rate g is superiorly and inferiorly limited because $0 \leq b_i \leq 1$.

In general, the model explains structural changes by means of workers' distribution movements between the two sectors. Indeed, changes in L_R (and L_C) reflect changes in b_i, which are the results of product and process innovations. We know that as long as new patents are produced by means of product innovations, b_i continues to increase over time implying a continuous shift of workers from the sector in which final consumption goods are produced to the innovative sector, with an increasing value of g_i. However, once there is a process innovation which reduces γ_i, changes in b_i are more complex and they explain structural changes of different nature, which may end up also with workers shifted from the innovative sector towards the sector in which consumption goods are produced if b_i for the new type of varieties is larger than it was for previous varieties on the frontier.

Particularly, we may state that there is a redistribution of workers from the innovative (final good) toward the final good (innovative) sector when the value which b_i takes once the process innovation takes place is smaller (larger) than its value for previous varieties on the frontier. In Appendix A we show that when process innovations are relatively not too big, b_i decreases after process innovations take place with workers moving from the innovative sector to the final good sector and, as a consequence, the growth rate of patents decreases.

Once the process innovation has taken place, as long as there are further innovations which increase the number of patents with the same value of γ, workers move from the final to the innovative sector. They are induced to move again to the final sector, once a subsequent process innovation of limited impact takes place.

Regarding the scale effect, we notice that it would still be present in this work if we had not introduced the assumption in (19) that increases in L may produce process innovations. These continuous subsequent process innovations due to increases in L may contribute to continuously lowering the value of b_i and keeping g from increasing.

In particular, this could not happen as long as subsequent patent innovations are related to varieties characterized by the same value of γ. In fact, partially differentiating (36) with respect to L_R, L and b_i, after few steps we obtain

$$\frac{dL_R}{L_R} = \frac{(1-\alpha)\, b_i}{(Lb_i\,(1-\alpha) - a\rho\alpha)} \left(L\frac{dL}{L} + \frac{\alpha\,(L + a\rho)}{((1-\alpha)\, b_i + \alpha)} \frac{db_i}{b_i} \right) \tag{38}$$

From (38) we know that L_R, and consequently g, would be constant only if

$$\frac{db_i}{b_i} = -\frac{L\,((1-\alpha)\, b_i + \alpha)}{\alpha\,(L + a\rho)} \frac{dL}{L} \equiv b^* < 0 \tag{39}$$

This is never the case when varieties of type i remain along the technological frontier given that b_i would continuously increase over time and, therefore, db_i/b_i can only be positive.

However, when L increases, continuous process innovations could continuously lower b_i. If these two effects on b_i balance each other, b_i will be constant implying that L_R, in turn, is constant with no change in the growth rate of the number of varieties. In appendix B we show that this would imply a constant growth of the real gross domestic product (GDP).

Therefore, we may conclude that when process innovations are associated to product innovations, we can obtain equilibrium paths characterized by a stable distribution of workers between the two sectors, which corresponds to a fixed growth rate, provided that b_i continuously decreases over time due to subsequent process innovations.

6 Conclusion

Scholars in the field of international economics and economic growth have devoted great attention to the subject of heterogeneity of firms in the last few years. Productivity differences across firms are, for instance, analyzed in a general equilibrium framework by Melitz (2003) which analyzes